**Optimal Control**

**Missile Guidance via LQG**

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Statement of the Problem:

We will consider an LQG problem in the realm of missile guidance in the simplified planar approximation case. The classical “guidance triangle” represents the relative geometrical configuration of the pursuer/target system.

Let denote the lateral position of the pursuer with respect to initial LOS, let denote the lateral velocity of the pursuer with respect to initial LOS, let denote the constant closing velocity (relative velocity of pursuer along the LOS). The range from pursuer to target is , where is the instantaneous time and is the final time. Let denote the angle between the initial LOS and the instantaneous LOS. is small when the pursuer is far from the target, and quickly grows towards the final time , when the pursuer is close to the target. For small enough :

Let denote the lateral acceleration of the pursuer – this is our input / control law.

Let denote the lateral acceleration of the target – this is a stochastic process, as the target is trying to evade the missile.

Both accelerations are with respect to the LOS.

Continuous Time Description:

The continuous time dynamics of the problem are:

Where is the control input as mentioned above, and is a random process, commonly modeled as a first order Markov process with the following equation:

Where is a known correlation time of the evader maneuvers and is a BM with intensity .

The initial lateral position, , is zero by definition. The initial lateral velocity, , is random. The statistical model:

This is according to the data given in the following section.

The measurement on-board the pursuer consists of the LOS angle , which is corrupted by an additive noise:

Where is BM with intensity:

The measurement equation can be written as:

Numerical Data:

Requirement:

Given the above assumptions, we need to find the optimal acceleration function , that minimizes the following performance index:

Where the admissible set for the acceleration is the set of functions of the history of the measurements:

Where denote the initial estimation covariances for the velocity and target acceleration, respectively.

Solution

First, we want to define a state vector , to fit the problem to the LQG pattern. Once we have the LQG pattern, we know how to solve it with partial information.

Let us define:

Also, we need out control input . Let us define:

Now, we want to get a differential equation of our model. To do this we will start with writing down and develop:

Now we find the matrices to reach the form of :

Finally, we get:

Where:

And as we know, is BM as mentioned above.

Next, we want to solve an LQG problem with partial information, with the measurement equation:

Which is given above.

The estimation equation is:

And from the lecture we’ve seen that is BM, and it's intensity is the same intensity as the BM , which is .

With this information in hand, we would want to minimize the cost with respect to the model of an LQR with full information on the estimation vector .

The requirement for our cost is:

According to the model mentioned, the solution to minimizing the cost with respect to the input is:

Where is the solution to the control Riccati equation:

Of course, in this case , so the equation is:

Kalman estimator:

Kalman gain:

Where in this case , and is the solution to the estimation Riccati equation:

With these sets of equations solved, we can get the optimal cost function:

What we do now is find the values of and then we need to discretize the model so we can simulate the problem in software. When doing the discretization, we want to set a time constant such that it is small enough to be able to react to the evasion maneuvers of the target, but not too small as to not increase the intensity of the covariance matrix , which is already increases as .

Since we know sec is the average time between maneuvers of the evader, and we have the requirement for sec, we want to make a decision faster than the average maneuver time, so we can choose half that time – 1sec, and then we get .

We then denote

Now we need to get the discrete model of the problem. We know that our continuous model is:

Where is BM with covariance .

Transforming to the discrete model will yield:

Where

And the covariance or the discrete noise is

The estimation model:

Where is BM with time variant covariance . The transformation to the discrete model:

In which case the covariance matrix for is .

We can see now why we do not want to be very small, as , even though it is discrete and has finite values, grows larger with the increase of . Dividing that by increases the intensity of the covariance even further, which causes the estimation error to grow as the missile gets closer to its target.

The next step is to discretize the cost, for small enough , we can convert the integral to a sum, and multiply by . This gives:

And since is constant it does not affect the minimization, so we need to minimize:

Where .

This is the problem we encountered as discrete LQG control:

Where are mentioned above intensities ( respectively), and .

With the discrete system we can go ahead and use dynamic programming to calculate the input to the system. To do this we need to calculate the Kalman gain and covariance matrix for each step:

Kalman gain:

Covariance matrix is calculated from the discrete estimation Riccati equation:

Feedback gain:

Where is calculated from the discrete control Riccati equation:

The state in this LQG problem is . Since we are trying to solve the LQG problem with partial information, we learned that we could use the estimated state to solve this problem as an LQG problem with full information, where the state is . To estimate , we use the Kalman gain the following way:

Then, with the feedback gain we can calculate our control input for each step:

Taking all the above into account, I programmed the process using MATLAB to get the input signal, cost and state at each interval .

The results of the simulation are shown below.





It is hard to see from the graphs, but the control signal is trying to steer the missile towards its target by applying control signals following Brownian motion. The estimation is very inaccurate at times close to , since we divide by the term . Hence, as we get closer to the magnitude of the control signal will increase to compensate, as we cannot see these changes in control direction on earlier times. The same problem occurs with the state vector as well.

As for the cost, we see that as expected, it will increase with each step. Since we have random noise in our process, we see an increase in cost at each time interval. For the same reason as above – dividing by – as we approach we can see the cost magnitude spiking dramatically.